Midterm 2 – Review – Problems

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1 Linear transformations and matrices

Problem 1:

Suppose V is a finite-dimensional vector space and $T \in \mathcal{L}(V, W)$ be invertible. Let $(v_1, \cdots, v_n)$ be a basis for V.

(a) Show that $(T(v_1), \cdots, T(v_n))$ is a basis for W

(b) What is the matrix A of T with respect to the basis $(v_1, \cdots, v_n)$ of V and $(T(v_1), \cdots, T(v_n))$ of W?

Problem 2:

Suppose A and B are $n \times n$ matrices such that $AB = I$. Show that $BA = I$.

Problem 3:

(3.13) Suppose V and W are finite-dimensional vector spaces and U is a subspace of V. Show that there exists $T \in \mathcal{L}(V, W)$ such that $Nul(T) = U$ if and only if $dim(U) \geq dim(V) - dim(W)$

2 Invariant subspaces and eigenstuff

Problem 4:

(Exercise 1 in Homework 6, with $n = 2$)

Suppose V is a finite-dimensional vector space and $T \in \mathcal{L}(V)$. Suppose $V = U \oplus W$, where U and W are $T-$invariant subspaces of V.

Let $(u_1, \cdots, u_m)$ be a basis for U and $(w_1, \cdots, w_k)$ be a basis for W.

Show that the matrix A of T with respect to the basis $(u_1, \cdots, u_m, w_1, \cdots, w_k)$ of V (you may assume it's a basis) is of the form:

$$A = \begin{bmatrix} B & O \\ O & C \end{bmatrix}$$

where B is a $m \times m$ matrix, and C is a $k \times k$ matrix!

Problem 5:

Suppose that W is a T-invariant subspace of V. Suppose that $v_1 + v_2$ is in W, where v_1 and v_2 are eigenvectors of T corresponding to distinct eigenvalues. Prove that v_1 and v_2 both belong to W.

Problem 6:

(if time permits, 5.3) Show that if U is a subspace of V that is invariant under every operator on V, then $U = \{0\}$ or $U = V$.